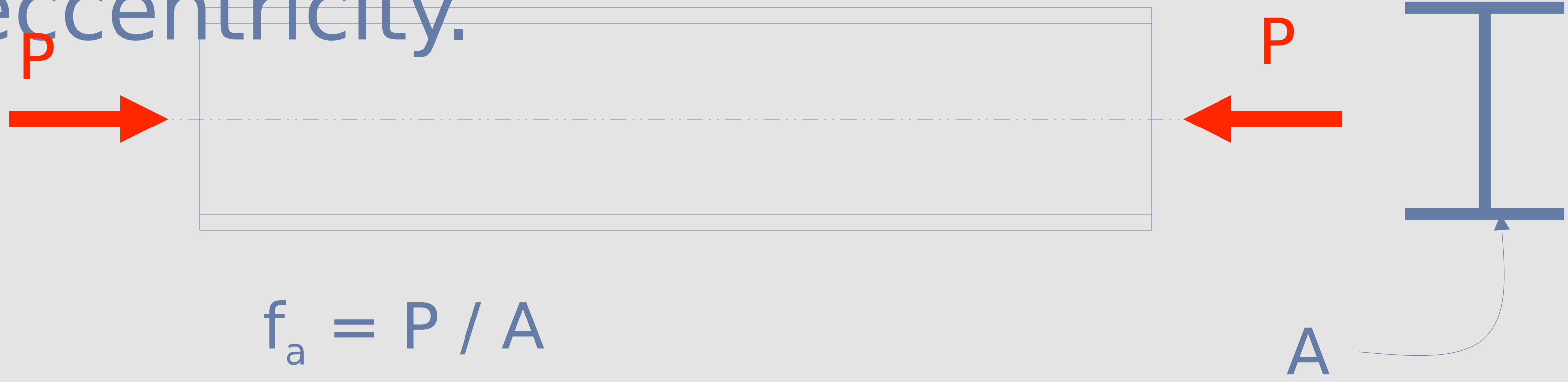
C & EE 141

Compression Members

Definition

- Elements which are subjected only to axial compressive forces are defined as **columns.**
- Loads are applied along the longitudinal centroidal axis with no eccentricity.



$$f_a = P / A$$

Idealized State of Concentric Loading

- The ideal state of concentric loading is never achieved. Some eccentricities always exist.
  - Members are not perfectly straight
  - Connections cause eccentricities
  - Internal stresses cause uneven stress distribution
- “Φ” factor accounts for inherent eccentricities.
- For significant eccentricities, we design a beam-column, which will be addressed in future lectures.

Common Compression Members

- Columns
- Braces
- Truss members
- Struts or Kickers

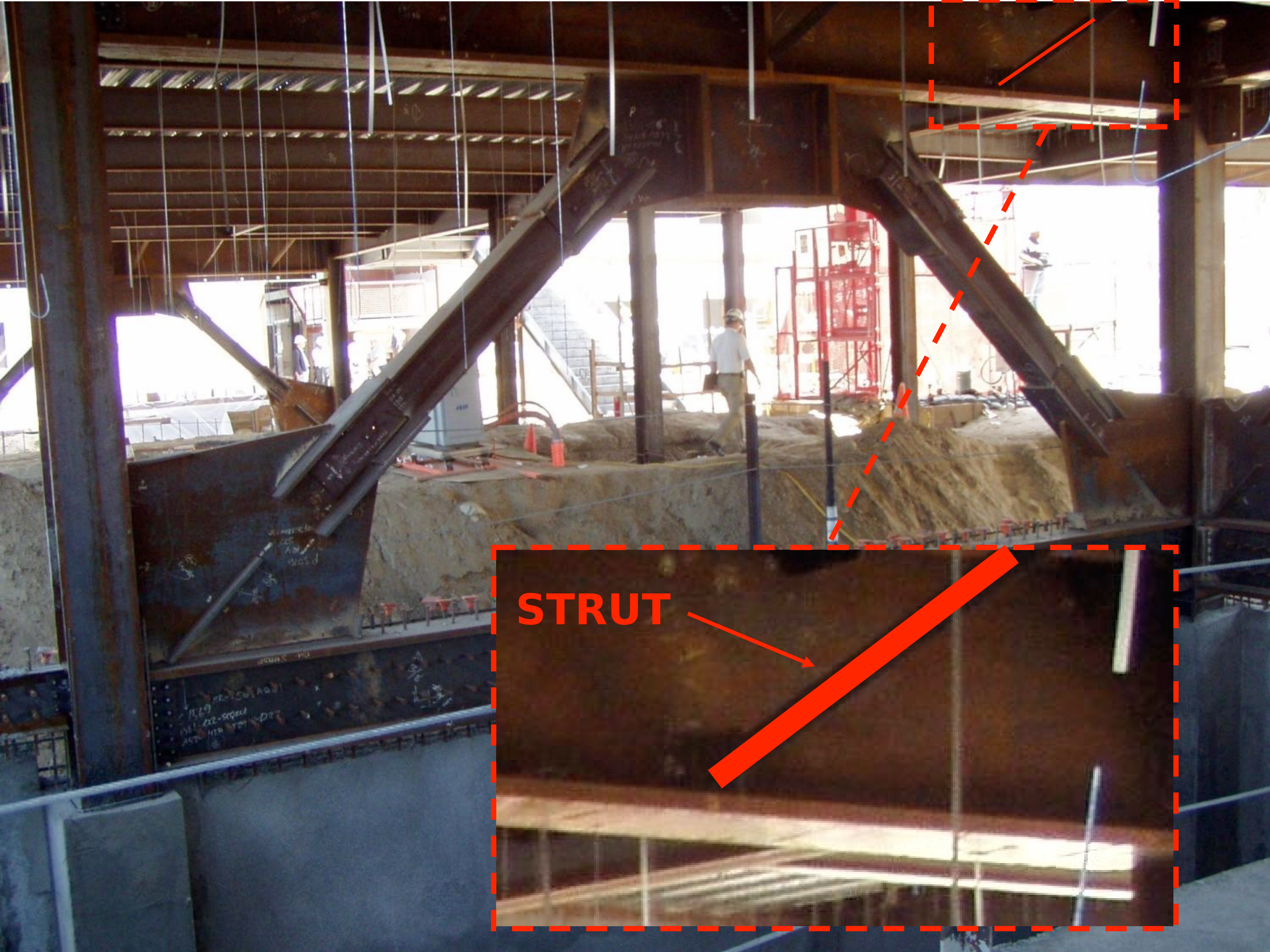

COLUMN


BRACES

Trusses

Members in Compression

Members in Tension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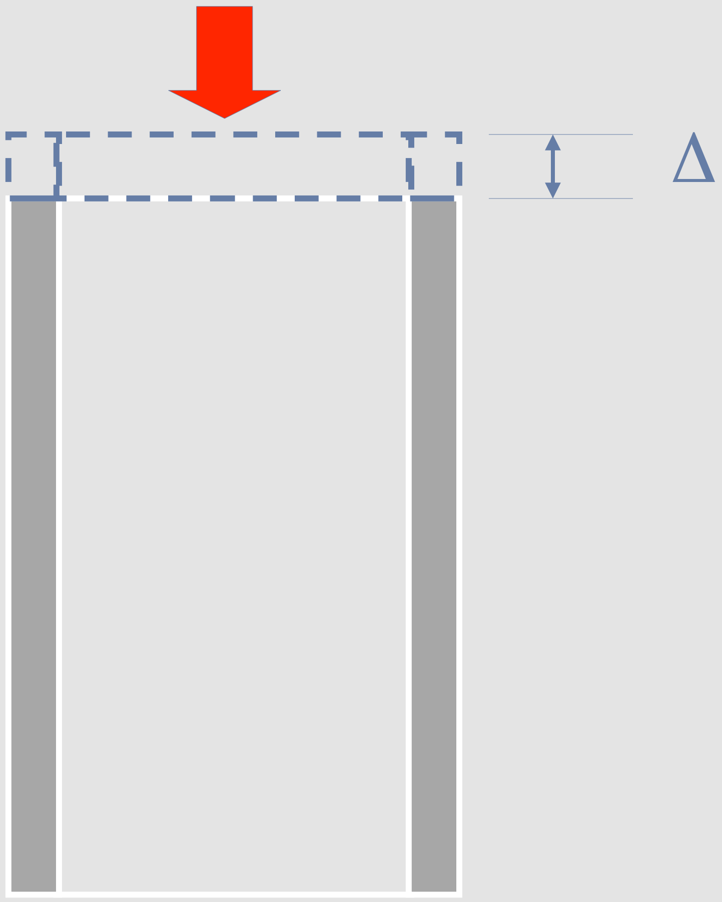
Column Limit States

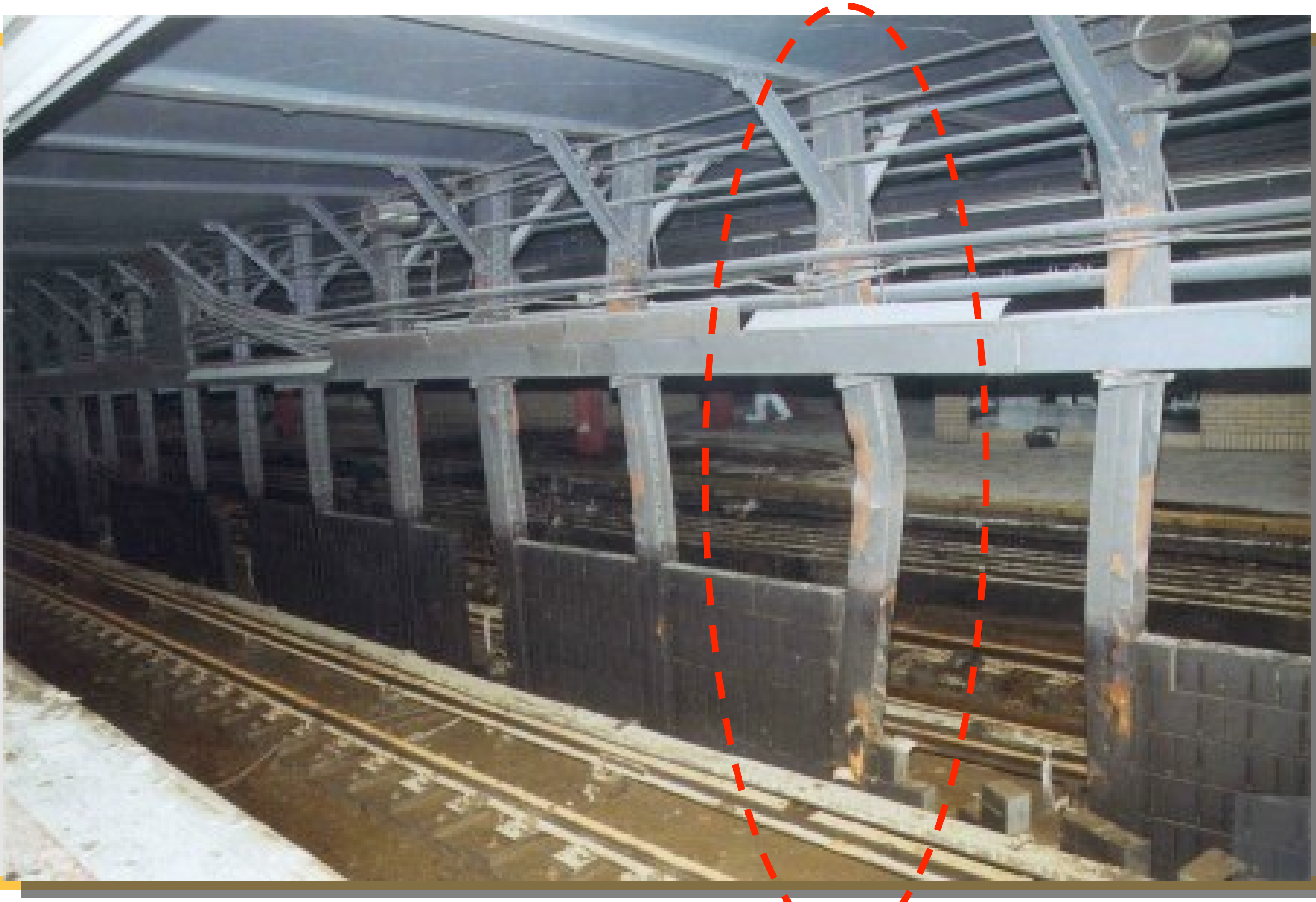
- Compression Yielding
  - Occurs only in very short or stout columns
- Global Buckling
  - Instability of the entire column
  - Strength is function of column length
- Local Buckling
  - Instability of a part of the column

Compression Yielding

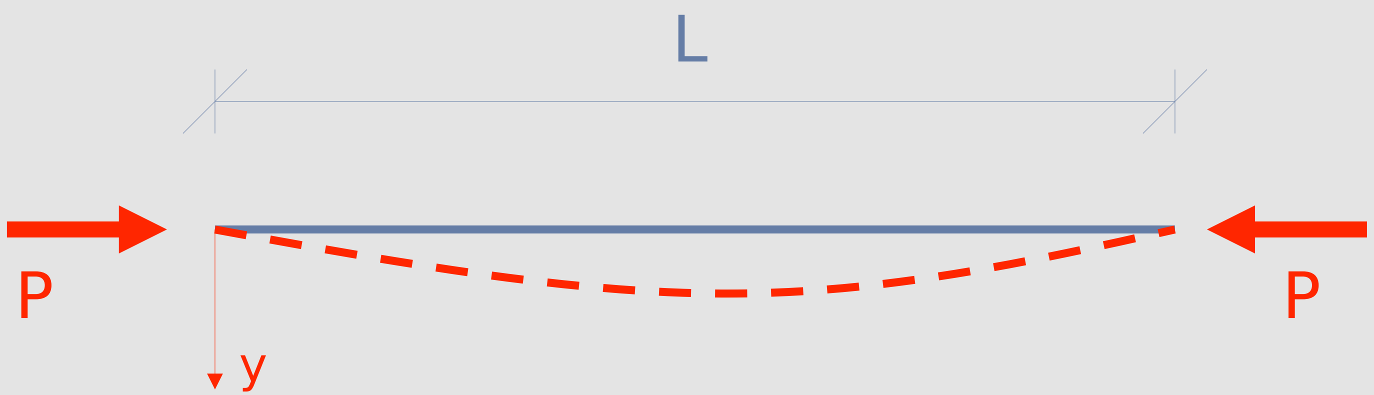
$$P_n = A_g F_y$$



Global Buckling



Column Theory



Fundamental Buckling Mode for a column in single curvature with pinned-end connections can be derived using differential equations. (Euler buckling—see Geschwindner Section 5.3).

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Column Theory (Cont.)

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Or in terms of average compressive stress:

$I = A_g r^2 \qquad r = \sqrt{I/A}$

$$F_{cr} = \frac{\pi^2 E}{\left(L/r\right)^2}$$

Critical buckling stress is a function of length, and radius of gyration (or, “slenderness ratio” L/r)

Elastic Buckling

- $$F_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$ (Euler Buckling)
- Problem: If L is very small, F could be infinite!
- But F_{max} must be F_y, the yield stress of the material (Compression Yielding).
- Another limit state, known as Inelastic Buckling, occurs which transitions from Elastic Buckling to

Inelastic Buckling

- What is Physically Occurring?
  - A combination of compression yielding and elastic buckling in different parts of the cross-section.
- Why?

Inelastic Buckling

- Residual stresses are present in W Shapes due to cooling. Like a pie, the edges cool faster than the inside. For a W shape, flange tips cool fast, and web-flange connections stay hot.

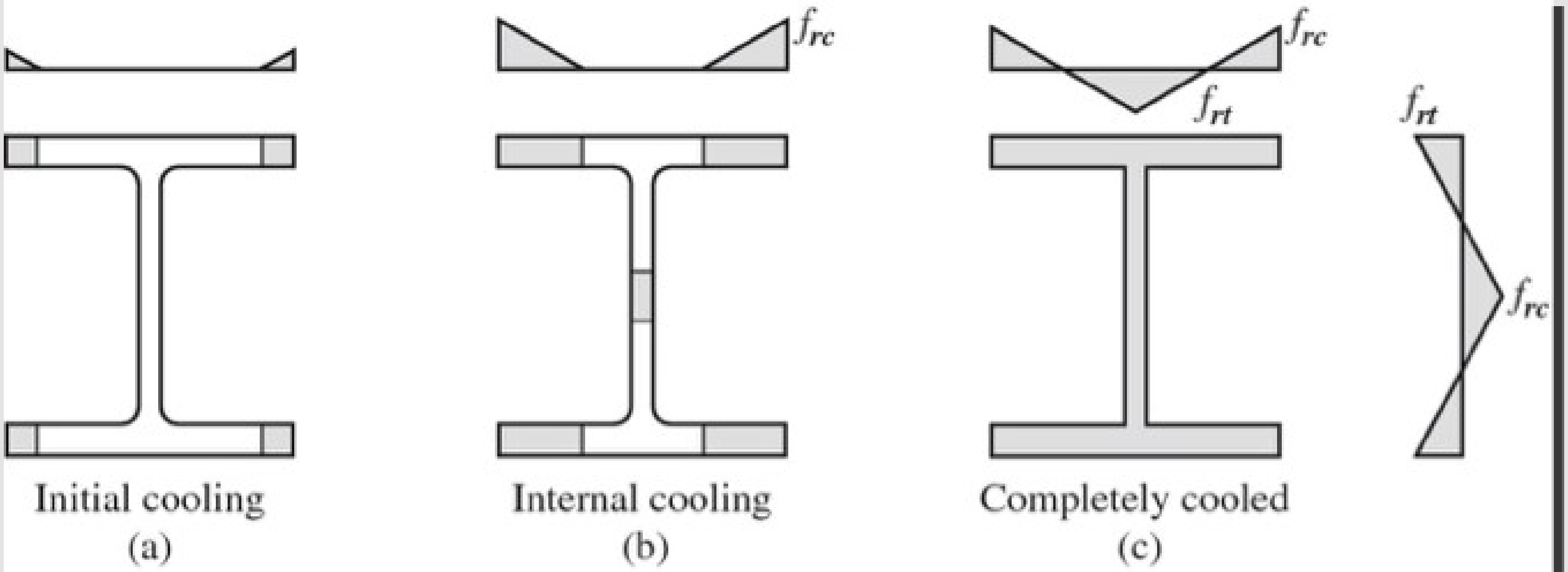


Figure 5.10

Inelastic Buckling

- This creates a “transition zone” between elastic and plastic behavior, between the proportional limit and

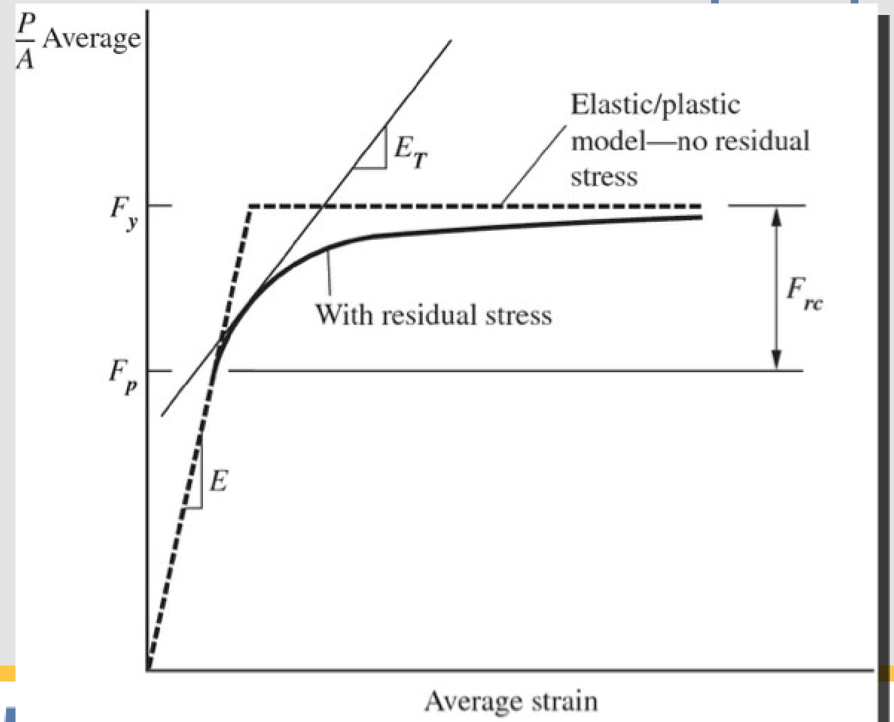
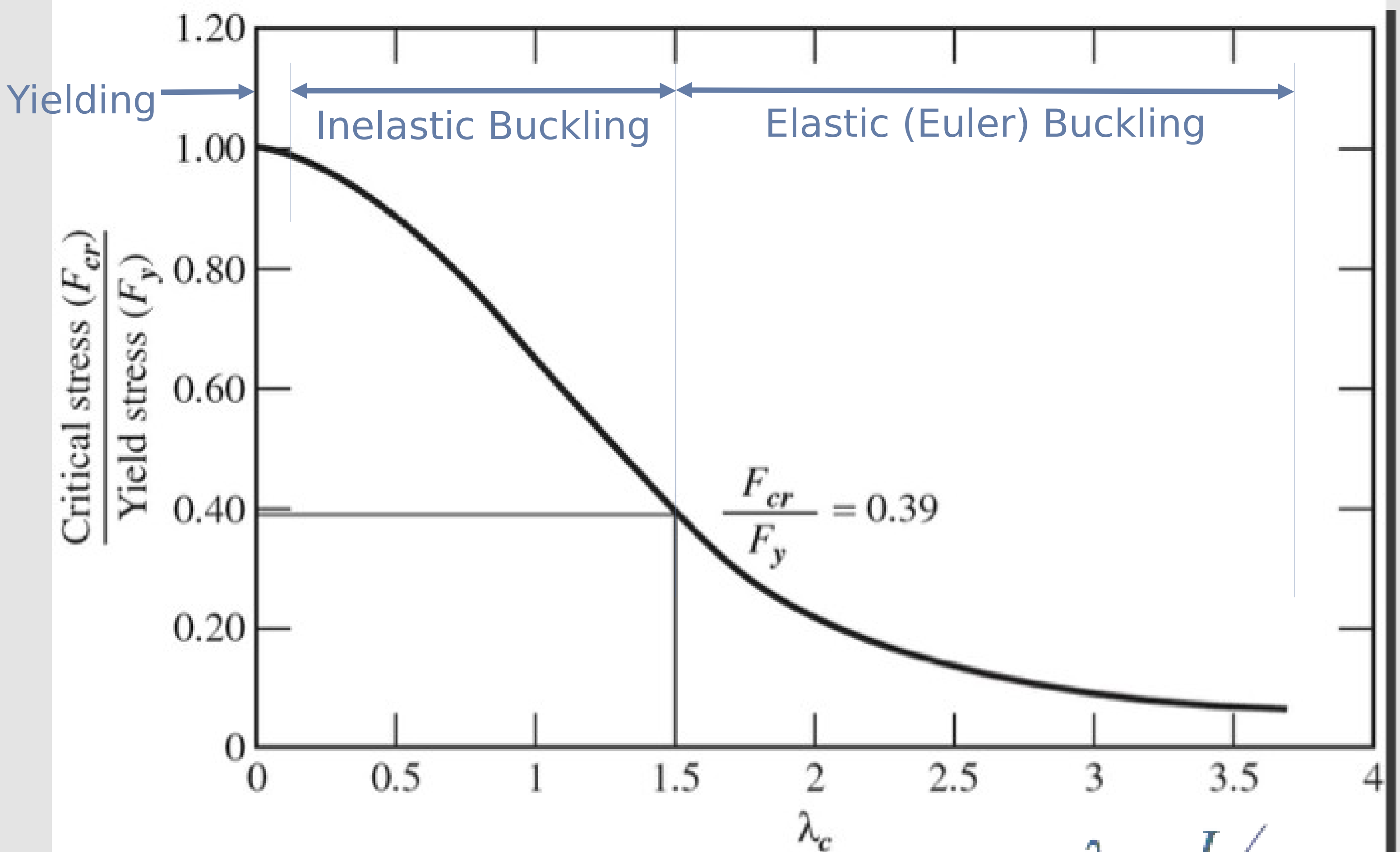


Figure 5.11

The equations are derived using a differential tangent modulus, E_t. (Engesser’s Solution)

$$\mathbf{P_{cr} = \pi^2 E_t I / L^2}$$

Transitions in Column Behavior



Note: $\lambda_c \propto L/r$

Effective Length

- Differential equations can also be solved for other boundary conditions
- We use a concept known as **effective length, K** to account for other boundary conditions.
- Understanding K:
  - Ask the question: What would the length be to “mimic” 1^{st} order buckling in a pin-pin connection?

Effective Length, K

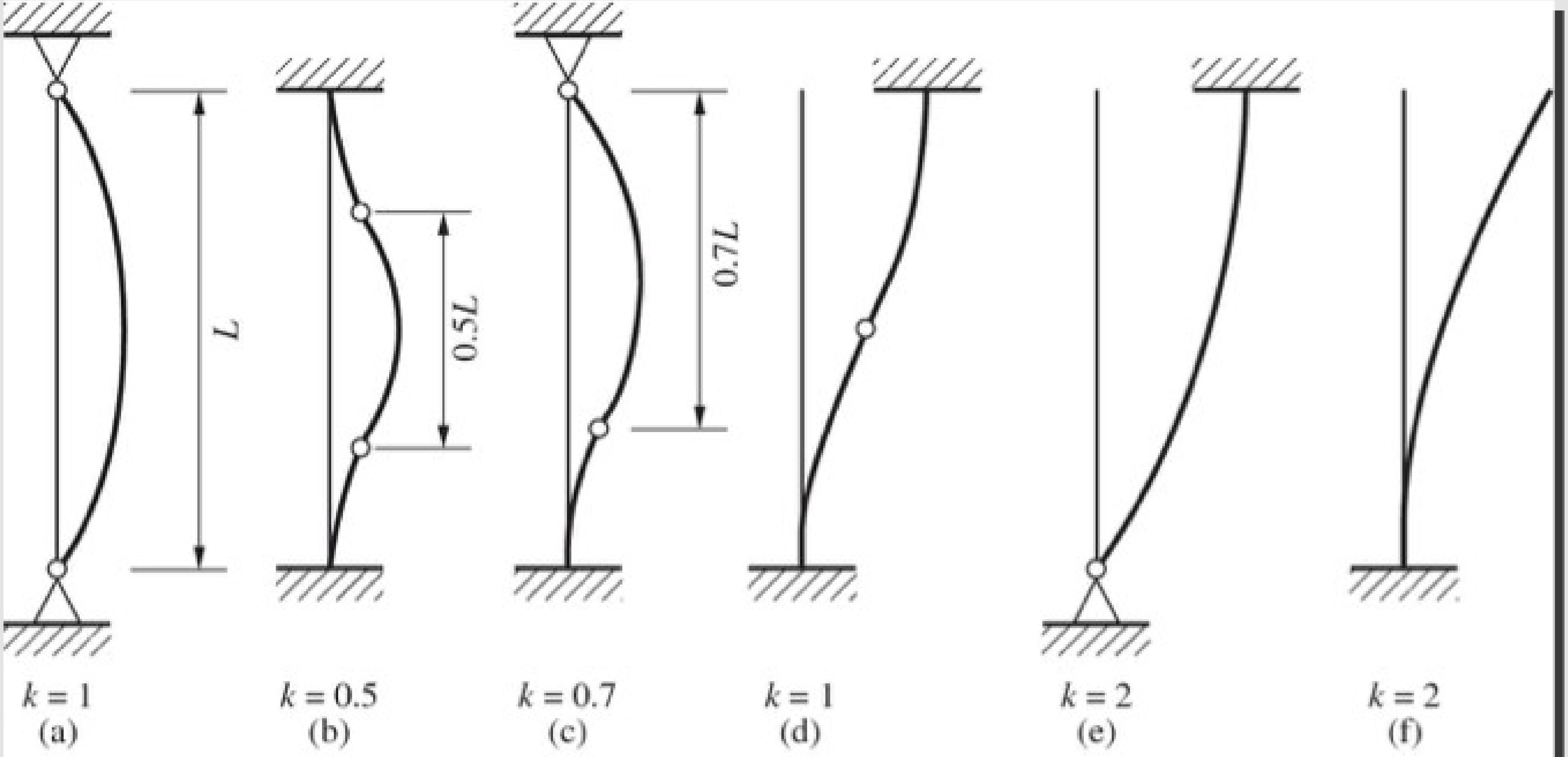


Figure 5.6

Effective Length, K

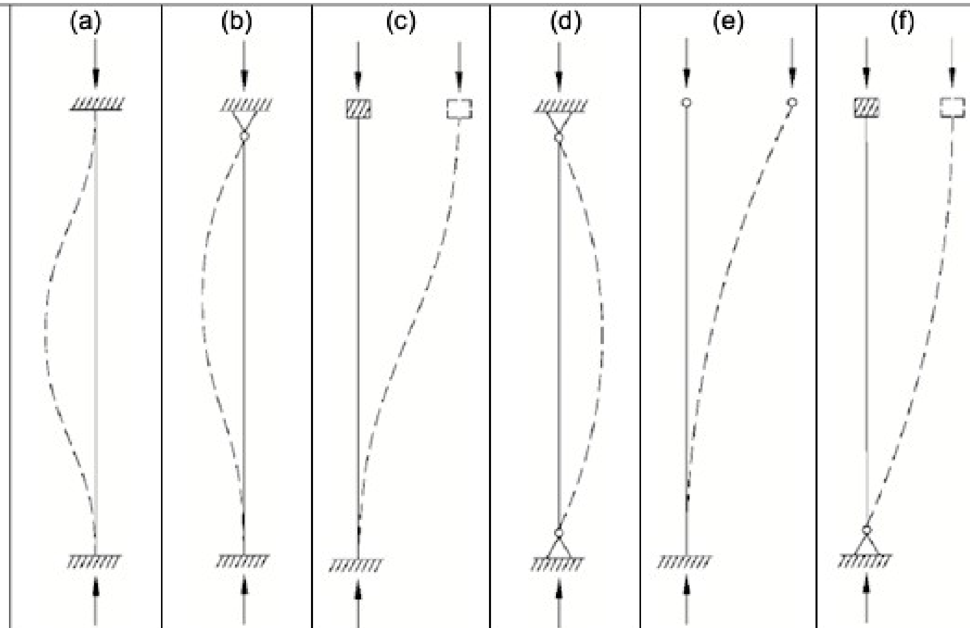
Rotation fixed and translation fixed

Rotation free and translation fixed

Rotation fixed and translation free

Rotation free and translation free

TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0

Axial Capacity

- $P_u \leq \phi_c P_n$
  - $\phi_c = 0.90$ (Section E1)
  - $P_n = F_{cr} A_g$ (nominal compressive strength)
  - F_{cr} (critical stress, Section E3)
    - $F_{cr} = [0.658^{\frac{F_y}{F_e}}]F_y$ for $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$ **INELASTIC BUCKLING**
    - $F_{cr} = 0.877F_e$ for $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$ **ELASTIC BUCKLING**
    - $F_{cr} \leq F_y$ **YIELDING**
    - Use largest value of KL/r for either x-x or y-y axis

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section applies to nonslender element compression members as defined in Section B4.1 for elements in uniform compression.

User Note: When the torsional *unbraced length* is larger than the lateral unbraced length, Section E4 may control the design of wide flange and similarly shaped *columns*.

The *nominal compressive strength*, P_n, shall be determined based on the *limit state* of *flexural buckling*.

$$P_n = F_{cr} A_g \tag{E3-1}$$

The *critical stress*, F_{cr}, is determined as follows:

(a) When $\dfrac{KL}{r} \le 4.71\sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} \le 2.25$)

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \tag{E3-2}$$

(b) When $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$ (or $\dfrac{F_y}{F_e} > 2.25$)

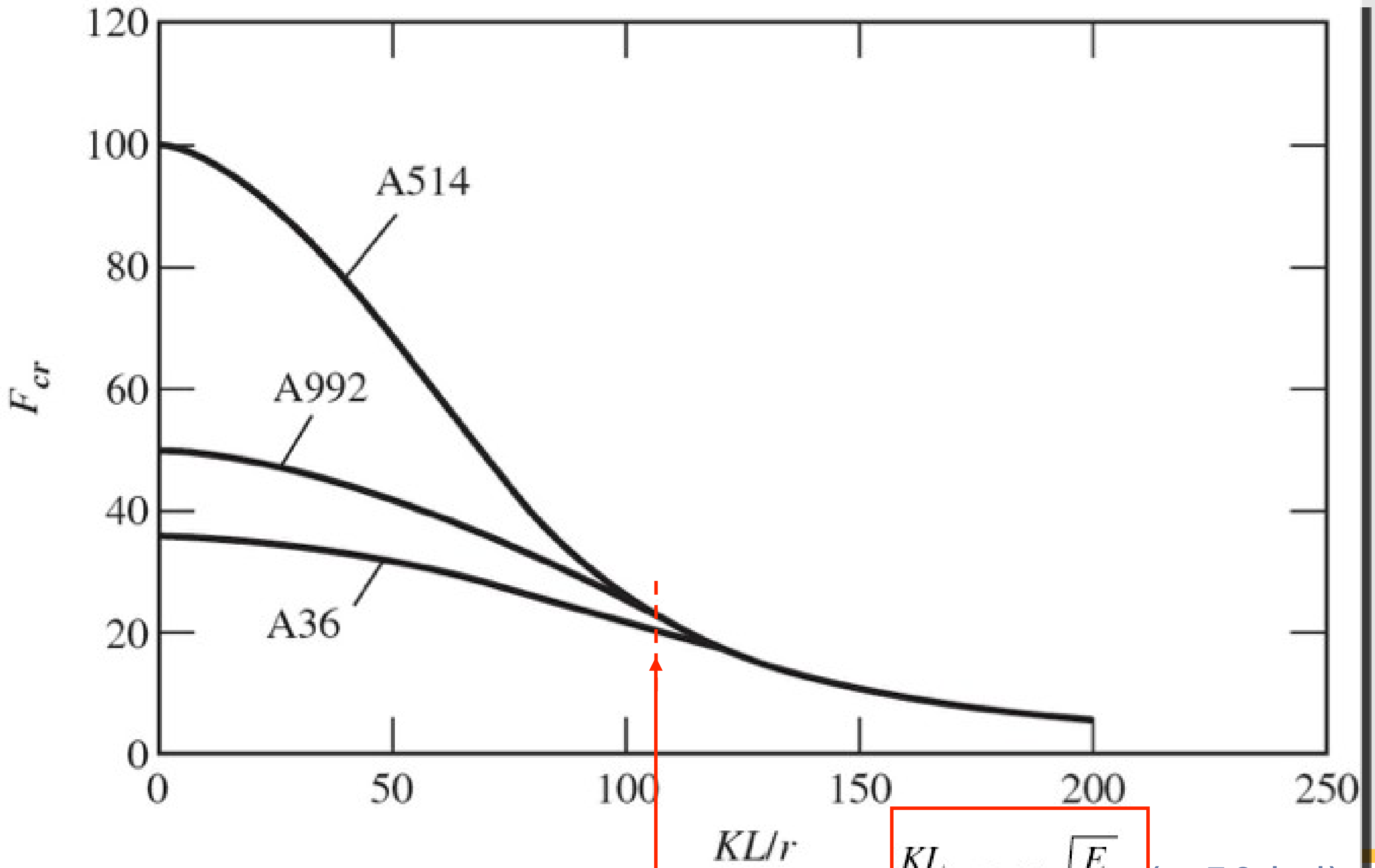
$$F_{cr} = 0.877F_e \tag{E3-3}$$

where

F_e = elastic *buckling* stress determined according to Equation E3-4, as specified in Appendix 7, Section 7.2.3(b), or through an elastic buckling analysis, as applicable, ksi (MPa)

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \tag{E3-4}$$

AISC Provisions



$$\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$$ (@50 ksi)

Figure 5.14

Maximum Slenderness Ratio

E2. EFFECTIVE LENGTH

The *effective length factor*, K, for calculation of member slenderness, KL/r, shall be determined in accordance with Chapter C or Appendix 7,

where

L = laterally *unbraced length* of the member, in. (mm)

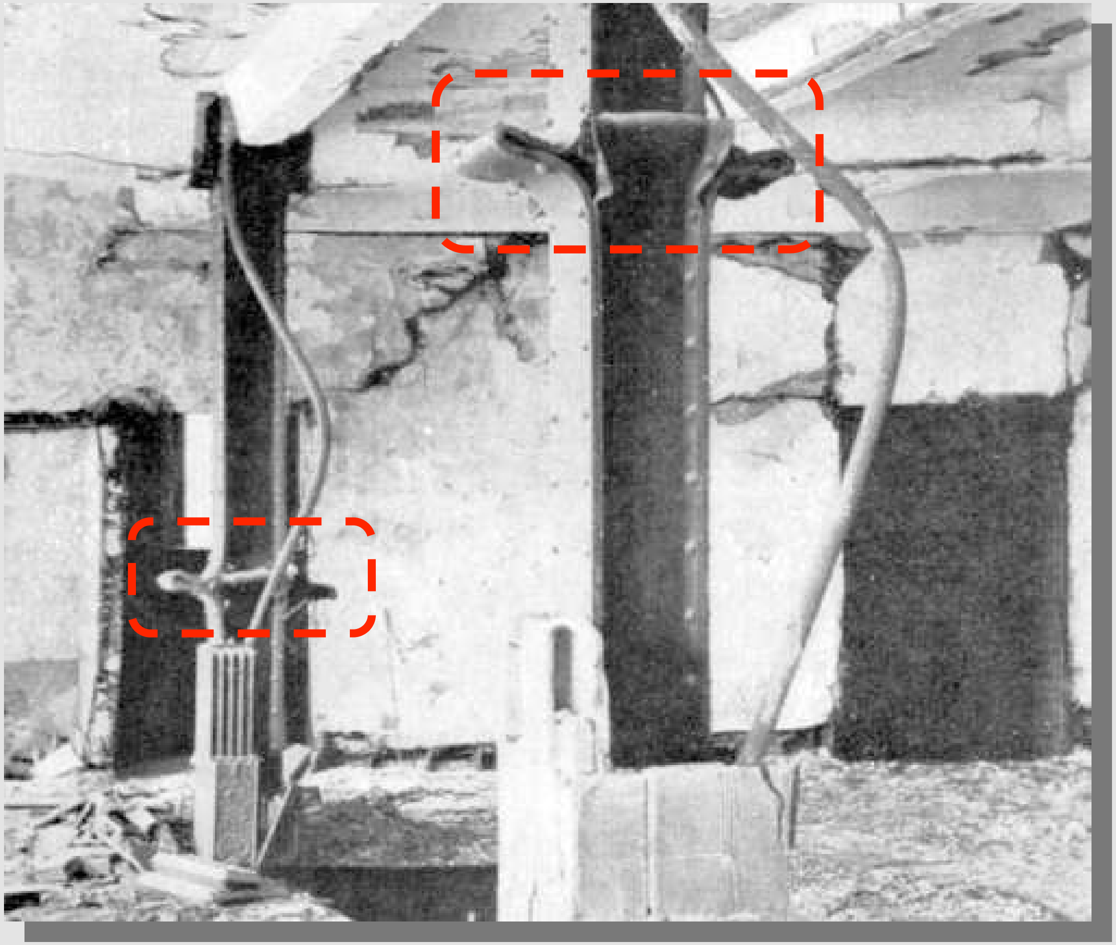
r = radius of gyration, in. (mm)

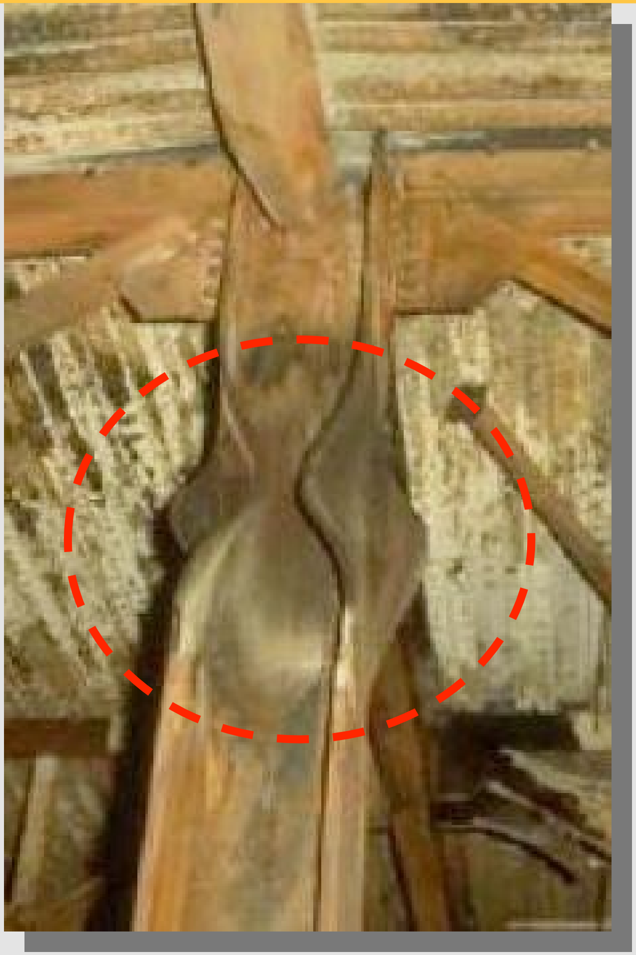
User Note: For members designed on the basis of compression, the effective slenderness ratio KL/r preferably should not exceed 200.

- Why?
  - At high slenderness ratios, critical stress is very low (~ 5 ksi)

Sidebar: Buckling About r_x or r_y?

Local Buckling

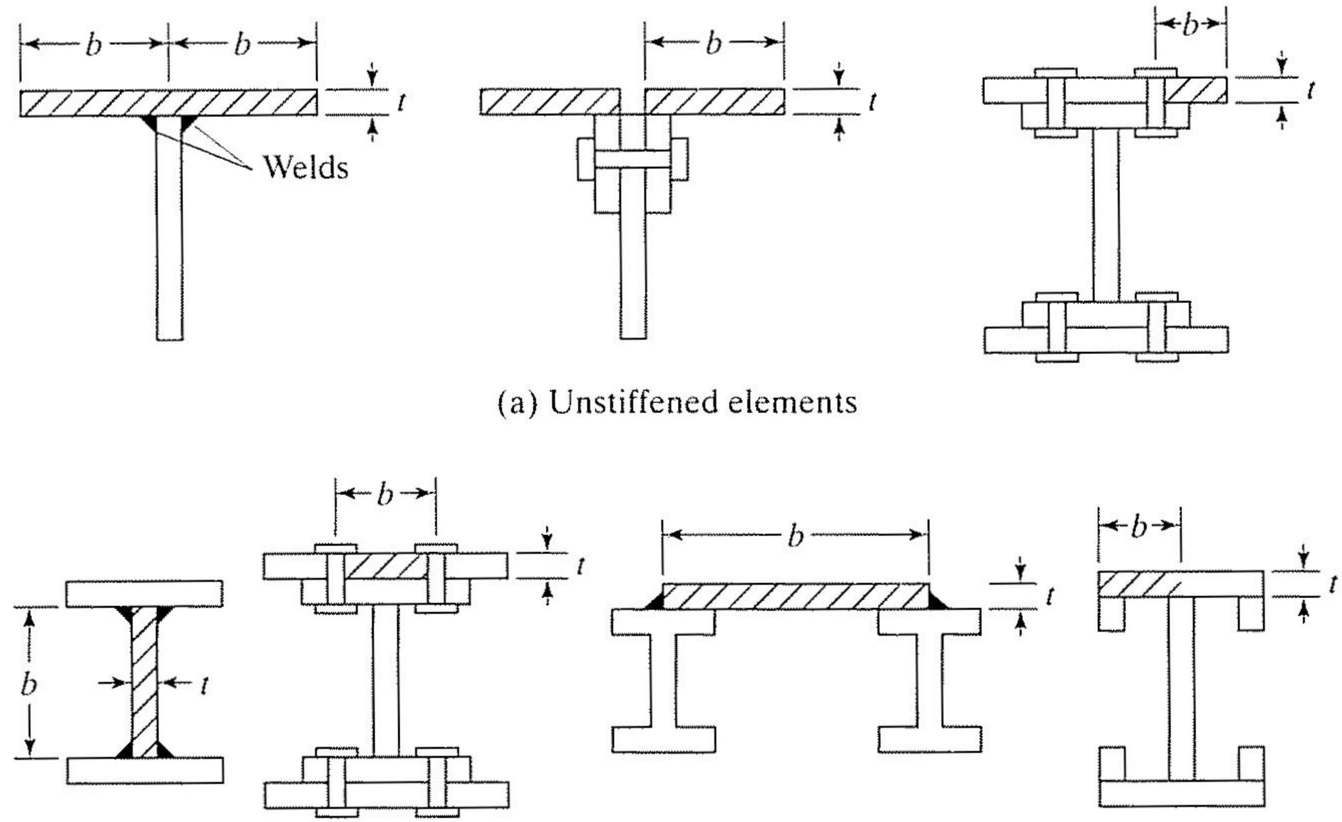




Local Buckling

- Stiffened Elements
  - A piece supported along two edges parallel to the direction of the compression force
- Unstiffened Elements
  - A piece with one free edge parallel to the direction of the compression force
- Key criterion is the width-thickness ratio
  (b/t ratio)

Stiffened & Unstiffened Elements



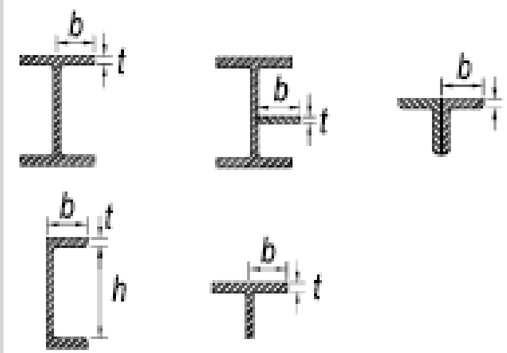
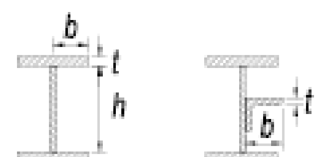
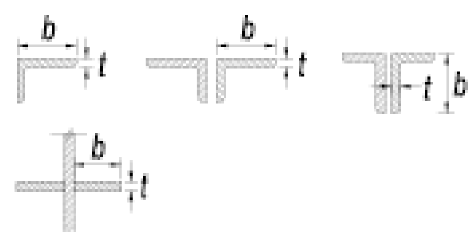
(a) Unstiffened elements

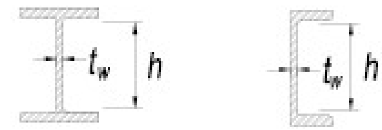
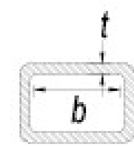
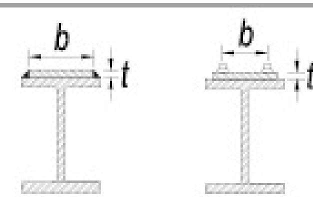
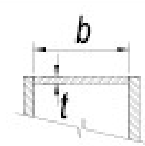
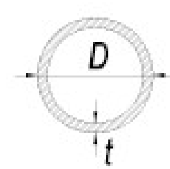
(b) Stiffened elements

Non-Slender Element vs. Slender-Element Sections

- Non-Slender Element Sections
  - Cross section is sufficiently "compact" so that global buckling will occur before local buckling of any cross-sectional elements
  - All elements of cross section must satisfy $b/t \leq \lambda_r$ from Spec Table B4.1a
- Slender-Element Section
  - Section with slender elements that may buckle locally before onset of global buckling
  - One that does not satisfy $b/t \leq \lambda_r$ requirement for one or more elements of the cross section
  - Must be checked per Spec Section E7 (complicated!)
- *Note: Terms "compact" and "non-compact" were used in the AISC 360-05 and previous standards. Terminology has been changed to clarify differing requirements for beams and columns.*

TABLE B4.1a
Width-to-Thickness Ratios: Compression Elements
Members Subject to Axial Compression

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender/slender)	Examples
Unstiffened Elements	1	Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections; outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees	b/t	$0.56\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	2	Flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t	[a] $0.64\sqrt{\frac{k_cE}{F_y}}$	
Unstiffened Elements	3	Legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	$0.45\sqrt{\frac{E}{F_y}}$	

Stiffened Elements	**5**	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$1.49\sqrt{\frac{E}{F_y}}$	
	6	Walls of rectangular HSS and boxes of uniform thickness	b/t	$1.40\sqrt{\frac{E}{F_y}}$	
	7	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.40\sqrt{\frac{E}{F_y}}$	
	8	All other stiffened elements	b/t	$1.49\sqrt{\frac{E}{F_y}}$	
	9	Round HSS	D/t	$0.11\frac{E}{F_y}$	

Nonslender Element vs. Slender-Element Sections

- Nearly all rolled sections are non-slender for compression, unless they are intended as beam shapes.
- Footnote in shape tables will alert you to slender-element sections.

W14×53	15.6	13.9	13 7/8	0.370	3/8	3/16	8.06	8	0.660	11/16	1.25	1 1/2	1	10 7/8	5 1/2
×48	14.1	13.8	13 3/4	0.340	5/16	3/16	8.03	8	0.595	5/8	1.19	1 7/16	1	↓	↓
×43[c]	12.6	13.7	13 5/8	0.305	5/16	3/16	8.00	8	0.530	1/2	1.12	1 3/8	1		
W14×38[c]	11.2	14.1	14 1/8	0.310	5/16	3/16	6.77	6 3/4	0.515	1/2	0.915	1 1/4	13/16	11 5/8	3 1/2[g]
×34[c]	10.0	14.0	14	0.285	5/16	3/16	6.75	6 3/4	0.455	7/16	0.855	1 3/16	3/4	↓	3 1/2
×30[c]	8.85	13.8	13 7/8	0.270	1/4	1/8	6.73	6 3/4	0.385	3/8	0.785	1 1/8	3/4		3 1/2

[c] Shape is slender for compression with $F_y = 50$ ksi.
[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.
[g] The actual size, combination and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.
[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Key Exclusions

- The provisions of Spec Section E3 consider the **flexural buckling** limit state.
- **Torsional buckling** and **flexural-torsional buckling** limit states also are possible, but we will not cover the details in the class.
- These provisions also assume that the compression element is not subject to local buckling prior to developing the strength noted. We must verify that the member is not **slender.**

Design Approach

- AISC SCM has a number of tables that facilitate the design:
  - Get to know PART 4
- Generally use Column Tables for design

Table for Critical Stress for Compression Members

Table 4-22

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

Table 4-22
Available Critical Stress for Compression Members

	$F_y = 35$ ksi			$F_y = 36$ ksi			$F_y = 42$ ksi			$F_y = 46$ ksi			$F_y = 50$ ksi	
$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	$\frac{KL}{r}$	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
1	21.0	31.5	1	21.6	32.4	1	25.1	37.8	1	27.5	41.4	1	29.9	45.0
2	21.0	31.5	2	21.6	32.4	2	25.1	37.8	2	27.5	41.4	2	29.9	45.0
3	20.9	31.5	3	21.5	32.4	3	25.1	37.8	3	27.5	41.4	3	29.9	45.0
4	20.9	31.5	4	21.5	32.4	4	25.1	37.8	4	27.5	41.4	4	29.9	44.9
5	20.9	31.5	5	21.5	32.4	5	25.1	37.7	5	27.5	41.3	5	29.9	44.9
6	20.9	31.4	6	21.5	32.3	6	25.1	37.7	6	27.5	41.3	6	29.9	44.9
7	20.9	31.4	7	21.5	32.3	7	25.1	37.7	7	27.5	41.3	7	29.8	44.8
8	20.9	31.4	8	21.5	32.3	8	25.1	37.7	8	27.4	41.2	8	29.8	44.8
9	20.9	31.4	9	21.5	32.3	9	25.0	37.6	9	27.4	41.2	9	29.8	44.7
10	20.9	31.3	10	21.4	32.2	10	25.0	37.6	10	27.4	41.1	10	29.7	44.7
11	20.8	31.3	11	21.4	32.2	11	25.0	37.5	11	27.3	41.1	11	29.7	44.6
12	20.8	31.3	12	21.4	32.2	12	24.9	37.5	12	27.3	41.0	12	29.6	44.5

Tables for Axial Capacity of Compression Members

Tables 4-1 through 4-20

W14

Table 4-1
Available Strength in Axial Compression, kips
W-Shapes

F_y = 50 ksi

Effective length, KL (ft), with respect to least radius of gyration, r_y

Shape	W14×											
lb/ft	730[h]		665[h]		605[h]		550[h]		500[h]		455[h]	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	6440	9670	5870	8820	5330	8010	4850	7290	4400	6610	4010	6030
11	6070	9130	5530	8310	5010	7530	4550	6840	4120	6200	3750	5640
12	6010	9030	5470	8220	4950	7440	4500	6760	4070	6120	3710	5570
13	5940	8920	5400	8110	4890	7350	4440	6670	4020	6040	3660	5500
14	5860	8810	5330	8010	4820	7250	4380	6580	3960	5950	3600	5420
15	5780	8690	5250	7890	4750	7140	4310	6480	3900	5860	3550	5330
16	5690	8560	5170	7770	4680	7030	4240	6380	3840	5770	3490	5240
17	5610	8430	5090	7650	4600	6920	4170	6270	3770	5660	3420	5150
18	5510	8290	5000	7520	4520	6790	4100	6160	3700	5560	3360	5050

Properties

	730 ASD	730 LRFD	665 ASD	665 LRFD	605 ASD	605 LRFD	550 ASD	550 LRFD	500 ASD	500 LRFD	455 ASD	455 LRFD
P_{wo}, kips	2820	4230	2410	3620	2060	3090	1750	2630	1500	2240	1280	1920
P_{wi}, kips/in.	102	154	94.3	142	86.7	130	79.3	119	73.0	110	67.3	101
P_{wb}, kips	44000	66100	34400	51700	26600	40100	20500	30800	15900	23900	12500	18800
P_{fb}, kips	4510	6780	3820	5750	3240	4870	2730	4100	2290	3450	1930	2900
L_p, ft	16.6		16.3		16.1		15.9		15.6		15.5	
L_r, ft	275		253		232		213		196		179	
A_g, in.^{2}	215		196		178		162		147		134	
I_x, in.^{4}	14300		12400		10800		9430		8210		7190	
I_y, in.^{4}	4720		4170		3680		3250		2880		2560	
r_y, in.	4.69		4.62		4.55		4.49		4.43		4.38	
r_x/r_y	1.74		1.73		1.71		1.70		1.69		1.67	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	409000		355000		309000		270000		235000		206000	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	135000		119000		105000		93000		82400		73300	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Table 4-4
Available Strength in Axial Compression, kips
Square HSS

HSS16-HSS14

F_y = 46 ksi

Effective length, KL (ft), with respect to least radius of gyration, r_y

Shape	HSS16×16× 1/2		HSS16×16× 3/8[c]		HSS16×16× 5/16[c]		HSS14×14× 5/8		HSS14×14× 1/2		HSS14×14× 3/8[c]	
t_{design}, in.	0.465		0.349		0.291		0.581		0.465		0.349	
lb/ft	103		78.5		65.9		110		89.7		68.3	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	780	1170	521	782	381	572	835	1250	678	1020	498	748
6	773	1160	518	779	379	570	825	1240	670	1010	494	743
7	770	1160	517	777	379	569	821	1230	667	1000	493	741
8	767	1150	516	776	378	568	817	1230	664	998	491	738
9	764	1150	515	774	377	567	813	1220	660	992	489	736
10	761	1140	513	772	376	566	808	1210	656	986	487	733
11	757	1140	512	769	375	564	802	1210	652	980	485	729
12	753	1130	510	767	374	563	796	1200	647	972	483	726
13	748	1120	508	764	373	561	790	1190	642	965	480	722
14	743	1120	506	761	372	559	783	1180	636	956	477	718
15	738	1110	504	758	371	557	775	1170	630	947	474	713

dius of gyration, r_y

Properties

Property	HSS16×16×1/2	HSS16×16×3/8	HSS16×16×5/16	HSS14×14×5/8	HSS14×14×1/2	HSS14×14×3/8
A_g, in.^{2}	28.3	21.5	18.1	30.3	24.6	18.7
$I_x = I_y$, in.^{4}	1130	873	739	897	743	577
$r_x = r_y$, in.	6.31	6.37	6.39	5.44	5.49	5.55

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

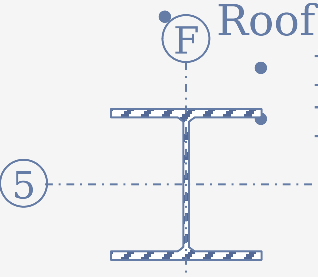
[c] Shape is slender for compression with $F_y = 46$ ksi.

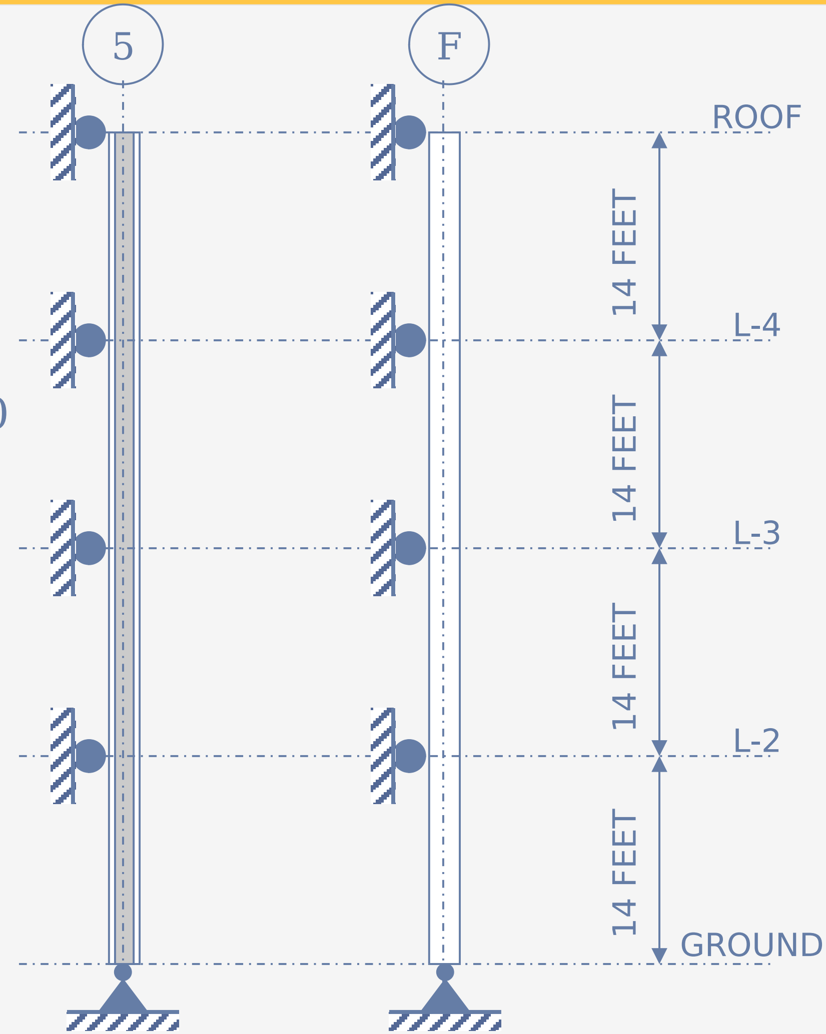
Questions?

Example Problem

Using only the design tables in the Steel Construction Manual, design the most efficient W10 wide flange steel column at Grid F/5 based on the conditions given below.

- The column is continuous from the ground level to the roof.
- The column supports a tributary area of 900 ft^2 at each floor.
- The loads on the column are as follows.
  - Level 2, 3 and 4
    - Dead Load = 120 psf
    - Live Load = 40 psf (non-reducible)
  - Roof
    - Dead Load = 100 psf
    - Live Load = 12 psf (non-reducible)





Example Problem

Five years after the building was constructed, the building owner wants to demolish the floor framing at Level-2 around the column at Grid F/5 to create a dramatic 28’-0” tall lobby. Your Project Manager proposes to weld Grade 50 (Fy = 50ksi) steel plates to the flanges to strengthen the column.

Determine if your Project Manager’s proposed solution is sufficient. Show all necessary calculations.